647510 **BSc4MatC401x** **Seat No :________**

B.Sc. Semester - 4 (*CBCS*) Examination

March/April-2026 [NEW COURSE]

Mathematics-M401(Core)

Time: 2:30 Hours **Marks: 70**

Instructions:

1. All questions are compulsory.
2. Figures to the right indicate marks.

Que.1(A) Answer any two. (10)

(1) Define: Monotonically increasing sequence. If $\{S_n\}$ is monotonically increasing and bounded above sequence then prove that $\{S_n\}$ is convergent and least upper bound of it, will be the limit of the sequence.

(2) State and prove Cauchy's first theorem on limits of the sequence.

(3) Discuss the convergence of the sequence, $S_1 = 4, S_{n+1} = 3 - \frac{2}{S_n}$; $n \in N$ and find its limit, if exists.

Que.1(B) Answer any one. (04)

(1) Prove that every convergent sequence is bounded.

(2) Prove that: $\lim_{n\to\infty} \sqrt[n]{n} = 1$.

Que.2(A) Answer any two. (10)

(1) Test the convergence of the series $1 + \frac{x}{2} + \frac{x^2}{5} + \frac{x^3}{10} + \cdots$

(2) State Raabe's test and discuss the convergence of the series

$$\frac{1}{2} + \frac{1\cdot 3}{2\cdot 4} + \frac{1\cdot 3\cdot 5}{2\cdot 4\cdot 6} + \cdots$$

(3) Using logarithmic test, test the convergence of series,

$$\frac{1^2}{4^2} + \frac{5^2}{8^2} + \frac{9^2}{12^2} + \frac{13^2}{16^2} + \cdots$$

Que.2(B) Answer any one. (04)

(1) Test the convergence of the series $\sum_{n=0}^{\infty} \frac{3^{2n}}{2^{3n}}$.

(2) Define: Alternating series. Show that that the series

$1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \cdots$ is convergent.

Que.3(A) Answer any two. (10)

(1) Let U and V be vector spaces over R. Prove that: $T: U \to V$ is linear transformation iff $T(\alpha u + \beta v) = \alpha T(u) + \beta T(v)$; $\forall \alpha, \beta \in R, \forall u, v \in U$.

(2) State and prove rank-nullity theorem for linear transformations.

(3) If $T: R^3 \to R^2, T(a,b,c) = (2a + b - c, 3a - 2b + 4c)$; $\forall (a,b,c) \in R^3$ is linear transformation and $B_1 = \{(1,1,1),(1,1,0),(1,0,0)\}$, $B_2 = \{(1,3),(1,4)\}$ are basis of R^3 and R^2 respectively then find a matrix $[T: B_1, B_2]$.

Que.3(B) Answer any one. (04)

(1) Prove that linear transformation $T: U \to V$ is one-one iff $N_T = \{\theta\}$. Where θ is the zero vector of U.

(2) If $S: U \to V$ and $T: U \to V$ are any two linear transformations then show that $S + T: U \to V, (S + T)(u) = S(u) + T(u)$; $\forall u \in U$ is also a linear transformation.

Que.4(A) Answer any two. (10)

(1) State and prove Cauchy-Schwartz's inequality for inner product space.

(2) Using Gram-Schmidt orthogonalization process, obtain orthonormal basis of R^3 with usual inner product from the basis $\{(1,0,1),(1,0,-1),(0,3,4)\}$.

(3) If u and v are vectors in a real inner product space then prove that:
$\|u+v\|^2 = \|u\|^2 + \|v\|^2$ iff $u \perp v$.

Que.4(B) Answer any one. (04)

(1) Let V be an inner product space. For $u, v \in V$ and any scalar α, prove that:
$(i)\|\alpha u\| = |\alpha|\|u\|$ $(ii)\|u\| \geq 0, \|u\| = 0 \iff u = \theta.$

(2) Define: Inner product space. If u and v are vectors in a real inner product space V then prove:
$(u+v)\cdot w = u \cdot w + v \cdot w.$

Que.5(A) Answer any two. (10)

(1) Derive the formula for radius of curvature for given parametric equations.

(2) Find the range of values of x for which the curve
$y = x^4 - 6x^3 + 12x^2 + 5x + 7$ is concave upwards or downwards. Find the point of inflexion in each case.

(3) If $y = mx + c$ is an asymptote to the curve $y = f(x)$ then prove that:
$m = \lim_{x\to\pm\infty}\left(\frac{y}{x}\right)$ and $c = \lim_{x\to\pm\infty}(y - mx).$

Que.5(B) Answer any one. (04)

(1) Define: (i) Double point (ii) Conjugate point or isolated point. Find the position and nature of the double points on the following curves:
$(i)(x^2+y^2)x - ay^2 = 0$ $(ii)y^2 = 2x^2y + x^3y + x^3.$

(2) Prove that the curvature of circle is constant and equal to the reciprocal of its radius.

****